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ON ESTIMATING AIR POLLUTION CONCENTRATIONS FROM A HIGHWAY IN AN OBLIQUE WIND

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Abstract—Concentrations produced by an extended near-ground line-source of vehicularly-generated highway air pollution are considered with special emphasis on the influence of the obliquity of the wind. A result previously quoted by Turner is clarified, and it is shown that over a wide range of wind directions the concentration distribution can be calculated by a simple approximate formula. Although the concentrations increase as the wind more closely parallels the direction of the highway, the increase is only slight and, for many practical purposes, could be disregarded.

INTRODUCTION

THE PRESENT vogue for mathematical modeling of air quality, based on calculation of atmospheric and diffusion, arises primarily because it provides a rational basis for the control of air pollution. Also, for land-use planning, modeling can provide estimates of potential hazards to air quality that may be expected from future planned developments. One aspect of the latter problem, currently receiving urgent attention, arises from the need to prepare environmental impact statements that estimate potential air pollution from proposed highway systems.

Perhaps the simplest, although quite important, problem of this type is that of the long straight highway located at ground level and passing through relatively unobstructed open countryside. Primary interest will normally be in the concentrations of vehicular-generated air pollution where these are greatest, i.e. at relatively short distances downwind from the highway. For such distances, the highway can often be treated sufficient accuracy as though it were of infinite length. In this case, the idealized problem considered in the simulation becomes the classical one of the concentration distribution from an infinite line source of matter. However, it is obviously desirable that this should be general enough to include situations where the wind is not perpendicular, but is oblique to the line source. Although the author is unaware of any theoretical analysis of this problem, a result for the oblique wind has previously been quoted by TURNER (1970). This is frequently used although the mathematical notation employed is ambiguous and has led to recent misinterpretation in applications. The present note provides a simple analysis of the problem and also aims to clarify the Turner result. It is further concluded that the pollutant concentration occurring at any given perpendicular distance from the highway is quite insensitive to the angle of the wind, and for many practical purposes can be regarded as independent of the latter even for very oblique winds.

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GENERAL ANALYSIS

The relative orientation of two sets of rectangular coordinate axes, Oxy and OXY, and the cross-section at ground level of the concentration plume from an imaginary, steady, continuous point-source located at the origin O are shown in FIG. 1.

The infinite line source coincides with the Y-axis and the wind direction is assumed to make an angle θ with the normal to the line. The axis Ox along the wind direction defines the axis of the point source cloud. The lines AA' and BB' pass through the axial point P distant x_o downwind from O and are, respectively, parallel to the axis Oy and OY.

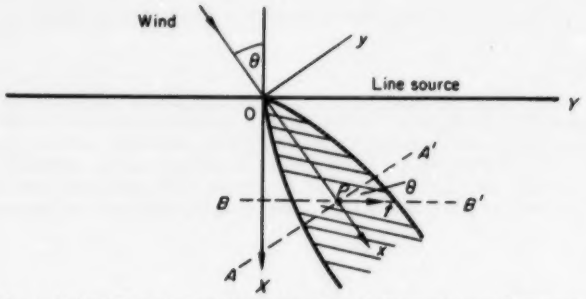

FIG. 1. Orientation of coordinate axes and point source plume (hatched).

Consider first the concentration C_p from an imaginary point source of constant strength Q_p at O. This will be written

$$C_p = Q_p \phi(x, y) \tag{1}$$

where the functional form of ϕ will not at first be specified. Let t denote distance along BB' from P (so that $x = x_o + t \sin\theta$, $y = t\cos\theta$) and $D(\theta, x_o)$ denote the integral of the point source concentration along the infinite line BB' (so that $D(0, x_o)$ is the integral along the infinite line AA'). Then

$$D(0, x_o) = Q_p \int_{-\infty}^{\infty} \phi(x_o, y)\, dy = say\, Q_p \psi(x_o) \tag{2}$$

and

$$D(\theta, x_o) = Q_p \int_{-\infty}^{\infty} \phi(x_o + t\sin\theta, t\cos\theta)\, dt. \tag{3}$$

However, unless the plume is very wide, the concentration gradient will be much greater normal to the wind direction than along it. Thus, over the limited width of the plume, the concentration variation for the oblique traverse along BB' will arise primarily from the variation of perpendicular distance from the plume axis rather than through the variation of distance parallel to this axis. In this case, equation (3) may be approximated by replacing the functional argument $(x_o + t\sin\theta)$ by x_o so that

$$D(\theta, x_o) \simeq Q_p \int_{-\infty}^{\infty} \phi(x_o, t\cos\theta)\, dt$$

$$= Q_p \int_{-\infty}^{\infty} \phi(x_o, y) \frac{dy}{\cos\theta} = Q_p \frac{\psi(x_0)}{\cos\theta}. \tag{4}$$

We may now proceed to the concentration distribution from the uniform infinite line-source along the Y-axis. For considering the line-source distribution as resulting from the superposition of plumes from infinitesimal point sources distributed along its length, it is evident that the line-source concentration at a location having perpendicular distance $X_o = x_o \cos\theta$ will be given by the point-source integrated concentration $D(\theta, x_o)$ above, provided Q_p is replaced by the line source strength Q_L per unit length of the line. Thus,

$$C_L(\theta, X_o) = Q_L \psi\left(\frac{X}{\cos\theta}\right)\Big/\cos\theta. \tag{5}$$

This is the basic result of the present analysis and indicates how the general concentration function ψ should be modified to allow for the obliquity of the wind. In the following section, we shall particularize and assume that ϕ is of the form appropriate to the well-known Gaussian plume for a point-source release. It may be noted that if the ψ-function can be represented by a simple inverse relation

$$\psi\left(\frac{X_o}{\cos\theta}\right) = \text{const} \times \left(\frac{\cos\theta}{X_o}\right) \tag{6}$$

then, from (5), $C_L(\theta, X_o)$ becomes independent of θ and varies inversely as X_o. This corresponds to the limiting case that is approximated by the example considered later in the numerical discussion (and for which $\psi(x_o) \sim 1/\sigma_z(x_o) \simeq x_o^{-0.865}$).

GAUSSIAN PLUME RESULTS

According to the Gaussian plume formula (see TURNER, 1970) the concentration at ground level from an elevated point source at height H (a non-zero H is included in formulae for sake of completeness only) corresponds, in equation (1) above, to

$$\phi(x, y) = \frac{\exp\left\{-\dfrac{y^2}{2\sigma_y^2(x)}\right\}\exp\left\{-\dfrac{H^2}{2\sigma_z^2(x)}\right\}}{\pi U \sigma_y(x), \sigma_z(x)} \tag{7}$$

where $\sigma_y(x)$ and $\sigma_z(x)$ are S.D. of the plume concentrations in the crosswind and vertical direction, respectively. These two functions must be specified in advance for the prevailing atmospheric conditions in terms of *ad hoc* observational data. It follows immediately from the integration of equation (2) that

$$\psi(x_o) = \frac{\sqrt{\left(\dfrac{2}{\pi}\right)}\exp\left\{-\dfrac{H^2}{2\sigma_z^2(x_0)}\right\}}{U\sigma_z(x_o)} \tag{8}$$

which thus specifies the form of the function that appears in the basic approximate line-source formula of equation (5). Hence, the approximate relation

$$C(\theta, X_o) = \frac{\sqrt{\left(\dfrac{2}{\pi}\right)}Q_L \exp\left\{-\dfrac{H^2}{2\sigma_z^2(X_o/\cos\theta)}\right\}}{U\cos\theta\cdot\sigma_z(X_o/\cos\theta)} \tag{9}$$

The Turner result (equation 5.19 as given in his Workbook and with his sin ϕ replaced by our cos θ) to be correct must be referred to the axes Oxy in FIG. 1. Unfortunately,

the functional argument of σ_z, which is quite critical, is not expressed emplicitly although when this is done Turner's formula becomes

$$X(x, y, 0; H) = \frac{\sqrt{\left(\frac{2}{\pi}\right)} Q_L \exp\left\{-\frac{H^2}{2\sigma_y^2(x)}\right\}}{U \cos\theta \cdot \sigma_z(x)} \tag{10}$$

where $X(x, y, 0; H)$ is used here to denote the concentration at position $(x, y, 0)$ for a line source at height H. However, with x held constant in equation (10), it is obvious that the quality represented by the left-hand side varies with y, in contrast to the right-hand side which is independent of y. The stated result will only be correct provided the left-hand side can be replaced by the special value $X(x, 0, 0; H)$. Although the notation used by Turner and his choice of coordinate axes do not appear to be appropriate for the line-source problem, the result obtained is then identical with equation (9) above. A more natural set of axes for the line-source problem appears to be the set we have denoted OXY, and Turner's formula of equation (10) above has sometimes been incorrectly interpreted relative to this set. With this interpretation, and now using our axis notation, equation (10) is replaced by the *incorrect* result

$$X(X_o, Y, 0; H) = \frac{\sqrt{\left(\frac{2}{\pi}\right)} Q_L \exp\left\{-\frac{H^2}{2\sigma_z^2(X_o)}\right\}}{U \cos\theta \cdot \sigma_z(X_o)} \tag{11}$$

Finally, the exact line-source result follows from equation (3) and is

$$C(\theta, X_o) = \frac{Q_L}{\pi U} \int_{-\infty}^{\infty} \exp\left\{-\frac{t^2 \cos^2\theta}{2\sigma_y^2(X_o/\cos\theta + t\sin\theta)}\right\} \exp\left\{-\frac{H^2}{2\sigma_z^2(X_o/\cos\theta + t\sin\theta)}\right\} dt \Big/ \sigma_y\left(\frac{X_o}{\cos\theta} + t\sin\theta\right) \sigma_z\left(\frac{X_o}{\cos\theta} + t\sin\theta\right). \tag{12}$$

For completeness, we also give the exact result for the singular situation with $\theta = 0$. This is easily seen to be

$$C_L(0, X_o) = \frac{Q_L}{\pi U} \int_0^{\infty} \frac{\exp\left\{-\frac{X_o^2}{2\sigma_y^2(\xi)}\right\} \exp\left\{-\frac{H^2}{2\sigma_z^2(\xi)}\right\}}{\sigma_y(\xi)\,\sigma_z(\xi)} d\xi. \tag{13}$$

When required, the definite integrals of (12 and 13) must be evaluated numerically.

SOME NUMERICAL COMPARISONS

A comparison between the concentration estimates based, respectively, on the use of the incorrect formula (11) and the formulae (9 and 12), is considered in this section. The calculations were made assuming that the atmospheric dispersion of the highway pollution is occurring under neutral conditions of atmospheric stability. For the

latter, it is assumed (GEOMET, 1971) that the crosswind standard deviation function can be approximately by

$$\sigma_y(x) = 0.13\, x^{0.903} \quad (x \text{ in m})$$

while the vertical S.D. $\sigma_z(x)$ was assumed to be of the form

$$\sigma_z(x) = a(x + c)^b$$

with the parameters a, b determined (with $c = 0$) so as to give a close fit to the well-known empirical Pasquill–Gifford diffusion curves. The values used were

$$\left.\begin{matrix} a = 0.0856 \\ b = 0.8650 \end{matrix}\right\} \text{for } x \leq 500 \text{ m}; \quad \left.\begin{matrix} a = 0.2591 \\ b = 0.6869 \end{matrix}\right\} \text{for } x \geq 500 \text{ m}.$$

A value was selected for c in an attempt to reflect the non-zero initial size of the pollution cloud produced by the traffic moving on the highway. For this purpose, it was assumed that $\sigma_z(0) = ac^b = 1.5$ m (corresponding to $c = 27$ m).

TABLE 1. CONCENTRATION ESTIMATES USING, TOP—INCORRECT FORMULA OF EQUATION (11): MIDDLE—APPROXIMATE FORMULA OF EQUATION (9): BOTTOM—"EXACT" FORMULA OF EQUATION (12)

X_o	$\theta = 0°$	$\theta = 15°$	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$	$\theta = 75°$
50 m	0.218	0.225	0.251	0.308	0.435	0.841
	0.218	0.221	0.231	0.250	0.282	0.338
	0.218	0.221	0.231	0.250	0.282	0.338
100 m	0.141	0.146	0.163	0.200	0.282	0.545
	0.141	0.143	0.148	0.156	0.171	0.197
	0.141	0.143	0.148	0.156	0.171	0.197
200 m	0.085	0.088	0.099	0.121	0.171	0.330
	0.085	0.086	0.088	0.092	0.099	0.121
	0.085	0.086	0.088	0.092	0.099	0.121
400 m	0.049	0.051	0.057	0.070	0.099	0.191
	0.049	0.050	0.051	0.054	0.061	0.076
	0.049	0.050	0.051	0.054	0.061	0.076
800 m	0.031	0.032	0.035	0.043	0.061	0.118
	0.031	0.031	0.032	0.043	0.038	0.048
	0.031	0.031	0.032	0.034	0.038	0.048

TABLE 1 shows values of the concentration estimates obtained using the three formulae; these values are for several different perpendicular distances from the highway and for a wide range of wind obliquity. (The calculations were made with Q_L and U both taken as unity and $H = 0$. Also, the integral of equation (12) was evaluated numerically by using the standard trapezoid rule based on 100 intervals.) All formulae, of course, give identical results when the wind is perpendicular to the highway. By inspection of the entries, the following conclusions may be drawn under the conditions assumed for these calculations.

(1) Based on the Gaussian plume dispersion formulation, concentration estimates derived using the simple approximate equation (9) are indistinguishable from those using the exact equation (12) that involves a tedious numerical integration.

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(2) At a given perpendicular distance from the highway, the concentration increases slightly as the wind direction more closely parallels that of the highway. However, this increase is only slight (i.e. roughly 50 per cent for varying from 0 to 75°) and for many practical purposes it should be possible to disregard the concentration dependency on obliquity of the wind.
(3) With very oblique winds ($\theta > 60°$) the incorrect formula that has sometimes been used gives values that differ appreciably from the correct ones.

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